

Upper critical field in superconductors near ferromagnetic quantum critical points; UCoGe

Yasuhiro Tada*, Norio Kawakami, and Satoshi Fujimoto

Department of Physics, Kyoto University, Kyoto 606-8502, Japan

We study the strong-coupling superconductivity near ferromagnetic quantum critical points, mainly focusing on the upper critical fields H_{c2} . Based on our simple model calculations, we discuss experimentally observed unusual behaviors of H_{c2} in a recently discovered ferromagnetic superconductor UCoGe. Especially, the large anisotropy between $H_{c2} \parallel a$ -axis and $H_{c2} \parallel c$ -axis, and the strong-coupling behaviors in $H_{c2}^{\parallel a}$ are investigated. We also examine effects of non-analytic corrections in the spin susceptibility on the superconductivity, which can arise from effective long range interactions due to particle-hole excitations.

KEYWORDS: superconductivity, upper critical field, ferromagnetism, quantum criticality, non-analytic corrections

1. Introduction

Recently, a new ferromagnetic superconductor UCoGe was discovered and has been attracting much interest.¹⁻⁸⁾ It is considered to be located near a ferromagnetic quantum critical point (QCP), and the related phenomena are extensively studied. In this compound, the superconductivity coexists with ferromagnetism in some pressure range, and the pairing state is expected to be spin triplet with equal-spin pairing along the easy axis of the magnetization, c -axis. The superconductivity extends to the paramagnetic phase and the superconducting transition temperature changes smoothly over the two phases, which suggests that the superconductivity is mediated by the Ising ferromagnetic spin fluctuations.⁹⁻¹¹⁾

Intensive experimental studies reveal interesting behaviors of the upper critical field H_{c2} which would be related to quantum criticality.^{2,4,5)} Naively, it may be expected that the anisotropy in the upper critical field H_{c2} in a superconductor with equal-spin pairing obeys $H_{c2}^{\parallel c} > H_{c2}^{\perp c}$, because there is no Pauli depairing effect for c -axis. In the coexistence region of the ferromagnetism and the superconductivity in UCoGe, however, the observed H_{c2} is $H_{c2}^{\perp c} \gg H_{c2}^{\parallel c}$, which is completely different from the naive expectation. Recently, it was pointed out that, for the superconductivity coexisting with the ferromagnetism which has a sufficiently large magnetization, the Pauli depairing effect for a field perpendicular to the magnetization does not destroy the superconductivity and the orbital depairing effect would be essential there.¹²⁾ Although this explains a very important part of the interesting behaviors of H_{c2} in UCoGe, there remain unresolved problems which would be beyond the weak-coupling theory. In addition to the unusual anisotropy, the behaviors of $H_{c2} \parallel \hat{a}, \hat{b}$ are surprising.^{4,5)} $H_{c2}^{\parallel a, b}$'s are huge exceeding 10 (T) while the transition temperature T_{sc} at zero field is 0.8 (K) and have upward curvatures, which would be considered as a characteristic property of the strong-coupling superconductivity. The huge H_{c2} with upward curvatures are also observed in noncentrosymmetric heavy fermion superconductors Ce(Rh/Ir)Si₃,^{13,14)} which can be understood as an interplay of lack of inversion symmetry and quantum critical-

ity.¹⁵⁾ For $H_{c2}^{\parallel b}$ in UCoGe, H_{c2} shows an S-shaped curve at high fields and the similarity to that in URhGe was pointed out.⁵⁾ These behaviors would be related to the ferromagnetic quantum criticality in UCoGe.

Apart from experiments, there have been new theoretical progress on ferromagnetic quantum criticality.¹⁶⁻¹⁹⁾ It is argued that the criticality can be affected by effective long range interactions due to particle-hole excitations which lead to the so-called non-analytic corrections. If the superconductivity is mediated by ferromagnetic spin fluctuations, it could be also affected by them.

In this study, we examine the strong-coupling superconductivity near ferromagnetic QCPs having UCoGe in mind. We restrict our calculations to the paramagnetic states and neglect the Pauli depairing effect. Under these conditions, we show that $H_{c2}^{\parallel a} \gg H_{c2}^{\parallel c}$ can hold in some parameter range. We perform the same calculations for two cases, the case of the point node symmetry and the line node symmetry, and conclude that the point node symmetry is a promising candidate for the superconductivity realized in UCoGe. We also study the effects of non-analytic corrections in the spin fluctuations on the superconductivity.

2. Calculations and Results

In this study, we use a very simple model to investigate the superconductivity near the ferromagnetic QCP.⁹⁻¹¹⁾ The action is

$$S = \sum_k c_k^\dagger [-i\omega_n + \varepsilon_k] c_k - \sum_q \frac{2g^2}{3} \chi(q) S_q^z S_{-q}^z \quad (1)$$

$$\chi(q) = \frac{\chi_0}{\delta + c_n q^2 \ln q + q^2 + |\Omega_n|/(vq)} \quad (2)$$

where $k = (i\omega_n, \mathbf{k})$, $c_k = (c_{k\uparrow}, c_{k\downarrow})^t$, $S_q^z = \sum_k c_{k+q}^\dagger (\sigma^z/2) c_k$, $\varepsilon_k = -2t \sum_{i=a,b,c} \cos k_i - \mu$, and the filling is fixed at $n = 0.15$ for which the Fermi surface is almost a sphere. We have taken the energy unit $t = 1$ and the length unit $a = 1$. The spin fluctuations are of Ising-type according to the NMR experiments⁸⁾ and the resulting pressure-temperature phase diagram can be consistent with the experiments.⁹⁻¹¹⁾ The criticality is characterized by $\delta = \delta_0(T + \theta) + c_h(\mu_B H_z)^2$, where we have just as-

*E-mail address: tada@scphys.kyoto-u.ac.jp

sumed the mean-field-like temperature dependence. Here, we fix $\delta_0 = 4$ and $v = 4$. The important point is that the spin fluctuations depend on the applied magnetic field along the c -axis H_z , as suggested by the resistivity measurements⁵⁾ and the NMR experiments.⁸⁾ The non-analytic correction is incorporated with the coefficient c_n which can arise from the effective long range interactions due to particle-hole excitations.^{16–19)} If we define the effective mass of the fluctuating modes as $\delta_{\text{eff}} \equiv \delta + \min[c_n q^2 \ln q + q^2]$, δ_{eff} becomes smaller than δ for $c_n > 0$, which means that the spin fluctuations are enhanced by the non-analytic correction. Here, we just put a remark on how actually the non-analytic correction can appear. In a fully SU(2) symmetric one-band model, all the scattering processes with momentum transfer $q \sim 0$ cancel out and cannot contribute to the spin susceptibility because of the loop cancellation theorem.²⁰⁾ For such a case, the non-analytic correction can arise from the scattering processes which include at least one scattering with $q \sim 2k_F$, where k_F is the Fermi wavenumber. On the other hand, the system with Ising-like symmetry, there would be no strong reason by which $q \sim 0$ scattering processes completely cancel out. This issue would be discussed in detail elsewhere.

To study the superconductivity focusing on the orbital depairing effect, we solve the Eliashberg equation without the Pauli depairing effect. The linearized Eliashberg equation reads,

$$\Delta_{\sigma\sigma}(k) = -\frac{T}{N} \sum_{k'} V(k, k') \mathcal{G}(k') \Delta_{\sigma\sigma}(k'), \quad (3)$$

$$\begin{aligned} \mathcal{G}(k) &= \frac{1}{2} [\langle \phi_0 | G(k + \Pi) G(-k) | \phi_0 \rangle \\ &\quad + \langle \phi_0 | G(k) G(-k + \Pi) | \phi_0 \rangle] \end{aligned} \quad (4)$$

where $\Pi = (0, -i\nabla_R - 2e\mathbf{A}(\mathbf{R}))$ with $\nabla \times \mathbf{A} = (H, 0, 0)$ or $(0, 0, H)$, and \mathbf{R} is the center of mass coordinate of the Cooper pair. ϕ_0 is the lowest level Landau function and G is the Green's function with the selfenergy evaluated at the lowest order in $g^2\chi_0$, $\Sigma(k) = (T/3N) \sum_q g^2\chi(q) G_0(k+q)$, where G_0 is the non-interacting Green's function. The pairing interaction is also calculated at the lowest order, $V(k, k') = -(1/6)g^2\chi(k - k') + (1/6)g^2\chi(k + k')$. The calculations are similar to those in the previous study.¹⁵⁾ We fix $t = 50$ (K) and $a = 4.0$ (Å) in this study, which corresponds to the effective mass of the cyclotron motion of the orbital depairing $m_{\text{eff}} \equiv \hbar^2/(ta^2) \simeq 440 \times (\text{bare electron mass})$.

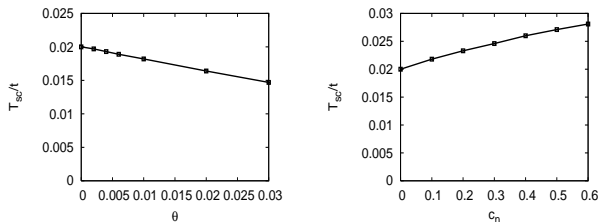


Fig. 1. (Left panel) Transition temperature T_{sc} as a function of θ for $c_n = 0$. (Right panel) T_{sc} as a function of c_n for $\theta = 0$.

First, we study the effects of the non-analytic correction in the spin fluctuations on the superconductivity at zero field. At zero field, the transition temperature for the gap function $d = (\sin k_a, \sin k_b, 0)$ with the point nodes and the

gap function $d = (\sin k_c, 0, 0)$ with the horizontal node are degenerate. In this study, the coupling constant is fixed as $g^2\chi_0 = 112.6t$ which is large so that we have rather high transition temperatures within our model. For this value of $g^2\chi_0$, the mass enhancement factor due to the ferromagnetic spin fluctuations is $z \sim 0.8$. The left panel of Fig. 1 shows the θ -dependence of the transition temperature T_{sc} which corresponds to the pressure-temperature phase diagram of the paramagnetic side in the experiment and $\theta = 0$ corresponds to the QCP. The transition temperature T_{sc} is highest at $\theta = 0$ and is decreased as θ increases, which is consistent with the experimental phase diagram of UCoGe.^{3,4)} In the right panel of Fig. 1, we show T_{sc} as a function of the coefficient of the non-analytic correction c_n . We see that T_{sc} is monotonically enhanced as c_n increases. This is because the effective mass of the criticality δ_{eff} becomes small with finite $c_n > 0$. This shows that the enhanced criticality by the non-analytic correction favors the superconductivity in the case of Ising-type spin fluctuations.

Next, we move to the effects of a magnetic field on the superconductivity and fix $\theta = 0$. In the presence of the magnetic field, we use the same model, eq.(2), although there might be other non-analytic corrections related to the magnetic field.^{19,21)} Even if we neglect Pauli depairing effects, the degeneracy of the symmetries of the gap functions is lifted by H and the anisotropy of H_{c2} depends on the positions of the gap nodes.²²⁾ This is simply explained as follows. The effective velocity for the cyclotron motion of the orbital depairing effect can be of the form of $\tilde{v}_k = \varphi(\mathbf{k})v_k$ for the basis function φ corresponding to the superconducting gap symmetry. When the magnetic field is parallel to the c -axis, the cyclotron motion can enjoy the nodes for the case with horizontal line nodes where $\tilde{v}_k = 0$, resulting in a large orbital limiting field. The cyclotron motion cannot do so for $H \perp \hat{c}$, which leads to the anisotropy of the orbital limiting field $H_{c2}^{\parallel c} > H_{c2}^{\perp c}$. On the other hand, under $H \parallel \hat{c}$, \tilde{v}_k cannot be zero for the point node gap function except for the poles of the Fermi surface, $k_a = k_b = 0$. Therefore, for point nodes at the poles of the Fermi surface, the order is turned over, $H_{c2}^{\parallel c} < H_{c2}^{\perp c}$.

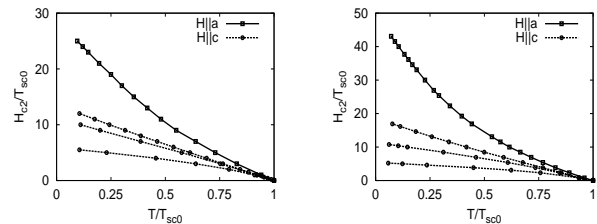


Fig. 2. Temperature v.s. upper critical fields H_{c2} for $c_n = 0$ (left panel) and $c_n = 0.4$ (right panel) for $d = (\sin k_a, \sin k_b, 0)$. The solid curves are for $H_{c2} \parallel a$ -axis, and the dotted curves are for $H_{c2} \parallel c$ -axis with $c_h = 0, c_h = 1.0, c_h = 10.0$ from the top to the bottom, respectively.

In Fig. 2, we show the upper critical field for several values of c_h for the point node case for which d -vector is $d = (\sin k_a, \sin k_b, 0)$. H_{c2} is normalized by the transition temperature at zero field T_{sc0} which is a measure of the Pauli limiting field for the usual s -wave superconductivity. The solid curve is for $H \parallel \hat{a}$ and dotted curves are for $H \parallel \hat{c}$. We see that, even if $c_h = 0$, $H_{c2}^{\parallel a} > H_{c2}^{\parallel c}$ holds because of the point

nodes. When the Ising spin fluctuations are suppressed by the magnetic field $H \parallel \hat{c}$ as suggested by the experiments,^{5,8)} $H_{c2}^{\parallel c}$ is reduced and it has only moderate temperature dependence for sufficiently large c_h . If there exists strong suppression of the spin fluctuations, H_{c2} can have strong anisotropy $H_{c2}^{\parallel a} \gg H_{c2}^{\parallel c}$. Note that, for the Ising anisotropic case, the spin fluctuations are robust against in-plane fields, resulting in that $H_{c2}^{\parallel a}$ remains large with the strong-coupling behaviors. The non-analytic correction can make the result more drastic and the calculated anisotropy is quite strong as seen in the right panel of Fig. 2.

The strong enhancement of the calculated $H_{c2}^{\parallel a}$ is understood as a result of an increasing pairing interaction and a decreasing depairing effect of the spin fluctuations at low temperatures. The physical origin is the same as that of the huge $H_{c2}^{\parallel c}$ in noncentrosymmetric heavy fermion superconductors Ce(Rh/Ir)Si₃.^{13–15)} Therefore, as pointed out in the previous work,¹⁵⁾ it can be said that the strong enhancement in the orbital limiting field near the quantum criticality has universal nature. In this study, however, we have assumed that the superconductivity is orbital limited, and the suppression of the Pauli depairing effect¹²⁾ would work well in the coexistence region of the ferromagnetism and the superconductivity in UCoGe. In the coexistence region, the mass of the criticality δ is replaced with the magnetization which has weak temperature dependence. Whether such a mass can actually lead to huge $H_{c2}^{\parallel a}$ without the assumption of the orbital-limited superconductivity is an open issue which should be addressed in the near future.

On the other hand, for the case of the gap function $d = (\sin k_c, 0, 0)$ with a horizontal line node, the relation $H_{c2}^{\parallel a} < H_{c2}^{\parallel c}$ holds when the suppression of the spin fluctuations is not so strong as shown in Fig. 3. For H_{c2} to be $H_{c2}^{\parallel a} > H_{c2}^{\parallel c}$,

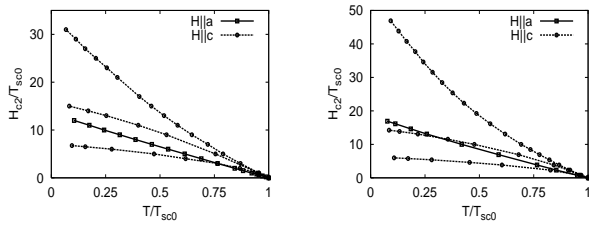


Fig. 3. Temperature v.s. upper critical fields H_{c2} for $c_n = 0$ (left panel) and $c_n = 0.4$ (right panel) for $d = (\sin k_c, 0, 0)$. The solid curves are for $H_{c2} \parallel a$ -axis, and the dotted curves are for $H_{c2} \parallel c$ -axis with $c_h = 0$, $c_h = 1.0$, $c_h = 10.0$ from the top to the bottom, respectively

a quite large c_h is needed, which cannot be usually realized.

From the above results, we conclude that a gap function with point nodes is more promising than that with horizontal line nodes as a candidate for UCoGe in the coexistence region. In UCoGe which has a orthorhombic structure, possible gap functions for the coexistent states are classified by the magnetic point group $D_2(C_2^z)$.²³⁾ Among the possible symmetries, the most promising candidate is the A_1 state or the A_2 state for H_{c2} to be consistent with the experiments.^{2,4,5)}

In summary, we have discussed the superconductivity near the ferromagnetic QCP within a simple model for the Ising spin fluctuations. The non-analytic correction can enhance

the superconducting transition temperature. Under a magnetic field, the positions of the nodes strongly affect the orbital cyclotron motion, leading to the anisotropy in H_{c2} . Taking into account the suppression of the Ising spin fluctuations by $H \parallel \hat{c}$, we have shown that the anisotropy can be $H_{c2}^{\parallel c} \ll H_{c2}^{\parallel a}$ for the point node case, and $H_{c2}^{\parallel a}$ has characteristic nature for strong-coupling superconductivity. On the other hand, it is rather hard to reproduce such behaviors for the horizontal line node case. These results suggest that the superconductivity realized in UCoGe is in the A_1 state or the A_2 state, both of which have no horizontal line nodes.

We thank K. Ishida, Y. Ihara, and D. Aoki for valuable discussions. Numerical calculations are partially performed at the Yukawa Institute. This work is partly supported by the Grant-in-Aids for Scientific Research from MEXT of Japan (Grant Nos. 19052003, 20102008, 21102510, and 21540359) and the Grant-in-Aid for the Global COE Program "The Next Generation of Physics, Spun from Universality and Emergence." N. K. is supported by JSPS through its FIRST program. Y. T. is supported by JSPS Research Fellowships for Young Scientists.

- 1) N. T. Huy, A. Gasparini, D. E. de Nijs, Y. Huang, J. C. P. Klaasse, T. Gortenmulder, A. de Visser, A. Hamann, T. Görlach, and H. v. Löhneysen: Phys. Rev. Lett. **99** (2007) 067006.
- 2) N. T. Huy, D. E. de Nijs, Y. K. Huang, and A. de Visser: Phys. Rev. Lett. **100** (2008) 077002.
- 3) E. Hassinger, D. Aoki, G. Knebel, and J. Flouquet: J. Phys. Soc. Jpn. **77** (2008) 073703.
- 4) E. Slooten, T. Naka, A. Gasparini, Y. K. Huang, and A. de Visser: Phys. Rev. Lett. **103** (2009) 097003.
- 5) D. Aoki, T. D. Matsuda, V. Taufour, E. Hassinger, G. Knebel, and J. Flouquet: J. Phys. Soc. Jpn. **78** (2009) 113709.
- 6) A. Gasparini, Y. K. Huang, N. T. Huy, J. C. P. Klaasse, T. Naka, E. Slooten, and A. de Visser: J. Low Temp. Phys. DOI 10.1007/s10909-010-0188-1.
- 7) K. Deguchi, E. Osaki, S. Ban, N. Tamura, Y. Simura, T. Sakakibara, I. Satoh, and N. K. Sato: J. Phys. Soc. Jpn. **79** (2010) 083708.
- 8) Y. Ihara, T. Hattori, K. Ishida, Y. Nakai, E. Osaki, K. Deguchi, N. K. Sato, and I. Satoh: arXiv:1008.2837.
- 9) P. Monthoux and G. G. Lonzarich: Phys. Rev. B **59** (1999) 14598.
- 10) Z. Wang, W. Mao, and K. Bedell: Phys. Rev. Lett. **87** (2001) 257001.
- 11) S. Fujimoto: J. Phys. Soc. Jpn. **73** (2004) 2061.
- 12) V. P. Mineev: Phys. Rev. B **81** (2010) 180504(R).
- 13) N. Kimura, K. Ito, H. Aoki, S. Uji, and T. Terashima: Phys. Rev. Lett. **98** (2007) 197001.
- 14) R. Settai, Y. Miyauchi, T. Takeuchi, F. Lévy, I. Sheikin, and Y. Ōnuki: J. Phys. Soc. Jpn. **77** (2008) 073705.
- 15) Y. Tada, N. Kawakami, and S. Fujimoto: Phys. Rev. Lett. **101** (2008) 267006.; Y. Tada, N. Kawakami, and S. Fujimoto: Phys. Rev. B **81** (2010) 104506.
- 16) D. Belitz, T. R. Kirkpatrick, and T. Vojta: Phys. Rev. B **55** (1997) 9452.; D. Belitz, T. R. Kirkpatrick and T. Vojta: Phys. Rev. Lett. **82** (1999) 4707.; D. Belitz, T. R. Kirkpatrick and J. Rollbühler: Phys. Rev. Lett. **94** (2005) 247205.
- 17) R. A. Duine and A. H. MacDonald: Phys. Rev. Lett. **95** (2005) 230403.
- 18) G. J. Conduit, A. G. Green, and B. D. Simons: Phys. Rev. Lett. **103** (2009) 207201.
- 19) A. V. Chubukov, C. Pépin, and J. Rech: Phys. Rev. Lett. **92** (2004) 147003.; D. V. Efremov, J. J. Betouras, and A. V. Chubukov: Phys. Rev. B **77** (2008) 220401(R); D. L. Maslov, A. V. Chubukov, and R. Saha: Phys. Rev. B **74** (2006) 220402(R).
- 20) W. Metzner, C. Castellani, and C. Di Castro: Adv. Phys. **47** (1998) 317.
- 21) S. Misawa: Phys. Rev. Lett. **26** (1971) 1632.; S. Misawa: Physica B **246-247** (1998) 382.
- 22) K. Scharnberg and R. A. Klemm: Phys. Rev. Lett. **54** (1985) 2445.
- 23) V. P. Mineev: Phys. Rev. B **66** (2002) 134504.; V. P. Mineev: J. Phys.

Soc. Jpn. **77** (2008) 103702.; V. P. Mineev: arXiv:0812.2171.